

PERSONAL INFORMATION

Joseph C. Várilly

📍 Universidad de Costa Rica, San José 11501, Costa Rica

📞 (+506) 8718-9674 📠 (+506) 2235-6578

✉ joseph.varilly@ucr.ac.cr

🌐 www.emate.ucr.ac.cr/josephvarilly

🆔 ORCID [0000-0001-9935-1123](https://orcid.org/0000-0001-9935-1123)

📅 **Date of birth** 20 February 1952 | 🇮🇪 **Nationality** Irish; resident of Costa Rica

CURRENT POSITION

Institute Universidad de Costa Rica, Escuela de Matemática
Rank Profesor Catedrático

ACADEMIC DEGREES

10/1972 B. Sc. in Mathematical Science, University College, Dublin
10/1973 M. Sc. in Mathematics, National University of Ireland
05/1975 M. A. in Mathematics, University of Rochester, NY, USA
05/1980 Ph. D. in Mathematics, University of Rochester, NY, USA

POSITIONS HELD

1977–78 Instructor, University of Rochester
1979–82 Visiting Professor, Universidad de Costa Rica
1982–84 Invited Professor, Universidad de Costa Rica
1984–87 Assistant Professor, Universidad de Costa Rica
1987–91 Associate Professor, Universidad de Costa Rica
1991 Visiting Professor, CInvEstAv del Instituto Politécnico Nacional, México, DF
1991–present Full Professor (Catedrático), Universidad de Costa Rica
1998–2005 Associate Member, International Centre for Theoretical Physics, Trieste, Italy
2005 Visiting Professor, Uniwersytet Warszawski, Warsaw, Poland
2007 Visiting Professor, Universidad Complutense de Madrid
2011 Visiting Professor, Universidad de Zaragoza

RESEARCH SUMMARY

Fields (with some overlaps):
1977–82 Operator algebras and applications.
1983–95 Phase-space quantum mechanics and applications.
1992–98 Noncommutative geometry and the Standard Model of particle physics.
1998–2013 Mathematical aspects of noncommutative geometry.
2009–12 Quantum chemistry.
2009–20 Quantum fields and string-local renormalization.

SCIENTIFIC CAREER

Summary My doctoral thesis and early work centred on problems in quantum statistical mechanics. Subsequently I have worked on problems in phase-space quantization (1985–91 and 2010–2012) and on noncommutative geometry and its physical applications (since 1992). I have also done some collaboration in marine biology modelling (1987). Currently I work on the string-local approach to quantum field theory.

- Books authored** On the teaching side, I have written a geometry text (1988, revised and enlarged in 2014) addressed to the needs of future mathematics teachers in Costa Rica, a monograph on Moyal quantization (1992), a graduate-level book on noncommutative geometry (2001, with two coauthors) and an introduction to noncommutative geometry (2006).
- Research areas** In the area of quantization, I have studied the structure and applications of the Moyal or quantum product in phase-space. In the early nineties, I turned to noncommutative geometry (in the line of Connes), with emphasis on its applications in particle physics and its behaviour under quantum symmetries. For a time, I retook phase-space methods with a view towards quantum chemistry. Recently, I have been working on causal and string-local quantum field theory.
- Lecture courses** I have given courses of lectures on noncommutative geometry (NCG) at several summer schools: at Monsaraz, Portugal (the European Mathematical Society Summer School on NCG) in September 1997; at São Paulo (the X Jorge André Swieca Summer School on Particles and Fields) in February 1999; at Villa de Leyva, Colombia (the CIMPA Summer School on Geometric and Topological Methods in Quantum Field Theory) in July 2001; at Nashville (the Clay Mathematics Institute Spring School on NCG and its Applications), in May 2003; and in Warsaw (as a Marie Curie TOK lecturer) during the fall semester of 2005–06.

RESEARCH VISITS

(of at least three weeks duration)

- 04/1976 – 12/1976 Instituto de Matematica, Universidade Estadual de Campinas, Brasil
Subject Infinite-dimensional holomorphy
- 11/1985 – 12/1985 International Centre for Theoretical Physics, Trieste, Italy
Subject Lie groups
- 01/1986 Departamento de Física Teórica, Universidad de Valladolid, Spain
Subject Phase-space quantum mechanics
- 11/1988 – 12/1988 Forschungszentrum BiBoS, Universität Bielefeld, Germany
Subject Phase-space quantum mechanics
- 06/1989 Departamento de Física Teórica, Universidad de Zaragoza, Spain
Subject Phase-space quantum mechanics
- 07/1991 – 02/1992 Centro de Investigación y Estudios Avanzados, IPN, México (on sabbatical)
Subject Phase-space quantum mechanics, noncommutative geometry
- 02/1993 – 03/1993 Forschungszentrum BiBoS, Universität Bielefeld, Germany
Subject Noncommutative geometry
- 01/1997 – 03/1997 Centre de Physique Théorique du CNRS, Marseille, France
Subject Noncommutative geometry and fundamental interactions
- 02/1999 Dipartimento di Matematica, Università di Bologna, Italy
Subject Noncommutative geometry
- 11/1999 International Centre for Theoretical Physics, Trieste, Italy
Subject Differential geometry
- 07/2000 – 08/2000 International Centre for Theoretical Physics, Trieste, Italy
Subject Noncommutative geometry
- 05/2001 Department of Mathematics, University of California, Berkeley, CA, USA
Subject Noncommutative geometry
- 09/2001 – 11/2001 International Centre for Theoretical Physics, Trieste, Italy
Subject Noncommutative geometry

- 02/2003 Dipartimento di Fisica, Università di Napoli Federico II, Napoli, Italy
Subject Noncommutative geometry and fundamental interactions
- 07/2003 – 10/2003 International Centre for Theoretical Physics, Trieste, Italy (on sabbatical)
Subject Noncommutative geometry and quantum symmetries
- 10/2003 – 12/2003 Institut des Hautes Études Scientifiques, Bures-sur-Yvette, France
Subject Noncommutative geometry
- 02/2004 Centre de Physique Théorique du CNRS, Marseille, France
Subject Noncommutative geometry
- 03/2004 Institut für Physik, Universität Mainz, Germany
Subject Noncommutative geometry and quantum symmetries
- 04/2004 – 05/2004 Fakultät für Physik, Universität Bielefeld, Germany
Subject Noncommutative geometry and quantum symmetries
- 07/2004 Department of Mathematics, University of Newcastle, NSW, Australia
Subject Noncommutative geometry
- 02/2005 – 04/2005 International Centre for Theoretical Physics, Trieste, Italy
Subject Noncommutative geometry and quantum symmetries
- 05/2005 Centre de Physique Théorique du CNRS, Marseille, France
Subject Noncommutative geometry
- 10/2005 – 01/2006 Katedra Metod Matematycznych Fizyki, University of Warsaw, Poland
Subject Dirac operators in noncommutative geometry
- 02/2007 – 04/2007 Departamento de Física Teórica, Universidad Complutense de Madrid, Spain
Subject Noncommutative geometry and quantum field theory
- 04/2008 Katedra Metod Matematycznych Fizyki, University of Warsaw, Poland
Subject Noncommutative geometry
- 10/2009 Institute of Mathematical Sciences, Australian National University, Canberra
Subject Noncommutative geometry
- 10/2010 Instytut Matematycznych Polskiej Akademii Nauk, Warsaw, Poland
Subject Dirac operators in noncommutative geometry
- 09/2011 – 12/2001 Departamento de Física Teórica, Universidad de Zaragoza, Spain (sabbatical)
Subject Quantum chemistry, quantum field theory
- 04/2013 – 05/2013 Fakultät für Physik, Universität Bielefeld, Germany
Subject Quantum field theory and causal renormalization
- 11/2015 Mathematisches Forschungsinstitut Oberwolfach, Germany
Subject Quantum field theory and string-local renormalization
- 10/2017 Instituto de Física Teórica UAM–CSIC, Madrid, Spain
Subject Quantum theory of string-local fields
- 03/2019 – 06/2019 Instituto de Física Teórica UAM–CSIC, Madrid, Spain
Subject “Invisible” quantum particles

CONFERENCE PARTICIPATION

- 05/1973 Conference on Infinite-Dimensional Holomorphy, Lexington, KY, USA.
- 05/1979 Conference on Operator Algebras, Rochester, NY, USA.

- 11/1980 III Costa Rican Physics Congress, San José, Costa Rica.
Talk: *Semigrupos dinámicos y ecuaciones de Bloch en sistemas abiertos finitos.*
- 11/1986 Workshop on Lie Groups and Representation Theory, ICTP, Trieste, Italy.
- 06/1989 XX International GIFT Seminar on Integrability and Quantization, Jaca, Spain.
- 11/1989 College on Differential Geometry, ICTP, Trieste, Italy.
Talk: *Geometric quantization on homogeneous symplectic manifolds.*
- 10/1990 Workshop on Differential Geometry and Mathematical Physics, CIMAT, Guanajuato, México.
Talk: *Moyal quantization on symplectic manifolds.*
- 03/1991 Workshop on Mathematical Physics and Geometry, ICTP, Trieste, Italy.
- 10/1991 Symposium on Hamiltonian Systems and Celestial Mechanics, CIMAT, Guanajuato, México.
- 11/1991 XXIV Congress of the Sociedad Matemática Mexicana, Oaxtepec, México.
Talk: *Representaciones proyectivas de grupos en la teoría cuántica de campos.*
- 06/1992 XIX International Colloquium on Group Theoretical Methods in Physics, Salamanca, Spain.
Talk: *Noncommutative geometry and the Standard Model: an overview.*
- 03/1993 Workshop on Mathematical Physics and Geometry, ICTP, Trieste, Italy.
Talk: *The infinite-dimensional spin representation and second quantization.*
- 06/1993 First Caribbean Spring School of Mathematics and Theoretical Physics, Saint-François, Guadeloupe, France.
- 02/1994 II Encuentro Centroamericano de Investigadores en Matemáticas, San Ramón, Costa Rica.
Talk: *La representación espín del grupo ortogonal infinitodimensional.*
- 03/1997 Colloque Géométrie Noncommutative et Intéractions Fondamentales, CIRM, Marseille, France.
Talk: *Spectral action from Cesàro asymptotics.*
- 03/1997 John Lighton Synge Centenary Conference, DIAS, Dublin, Ireland.
- 09/1997 EMS Summer School on Noncommutative Geometry and Applications, Monsaraz, Portugal.
Lecture series: *Introduction to noncommutative geometry.*
- 09/1997 Conference on Recent Results in Noncommutative Geometry, Lisboa, Portugal.
- 08/1998 Meeting on Noncommutative Geometry, Mathematisches Forschungszentrum Oberwolfach, Germany.
Talk: *Quantization fields over noncommutative manifolds.*
- 08/1998 International Congress of Mathematicians, Berlin, Germany.
Poster: *Quantum fields over noncommutative tori.*
- 02/1999 X Jorge André Swieca Summer School of the Sociedade Brasileira de Física, Aguas de Lindoia, SP, Brasil.
Lecture series: *Noncommutative geometry and quantization.*
- 12/2000 XI Simposio sobre Matemáticas, Ciencia y Sociedad, San José, Costa Rica.
Talk: *La geometría en su contexto histórico.*
- 04/2001 Workshop on Quantization and Noncommutative Geometry, Mathematical Sciences Research Institute (MSRI), Berkeley, CA, USA.

- Talk: *Quantum symmetry of noncommutative spheres.*
- 05/2001 29th Canadian Annual Symposium on Operator Algebras, Mathematical Sciences Research Institute (MSRI), Berkeley, CA, USA.
- 07/2001 CIMPA Summer School on Geometric and Topological Methods in Quantum Field Theory, Villa de Leyva, Colombia.
Lecture series: *Cyclic cohomology, Hopf algebras and quantum theory.*
- 09/2001 Conference on Noncommutative Geometry and Quantum Groups, Banach Center, Warszawa, Poland.
Talk: *Quantized symmetry groups of noncommutative homogeneous spaces.*
- 05/2002 VI Conference on Clifford Algebras and their Applications in Mathematical Physics, Cookeville, TN, USA.
Invited talk: *The interface of noncommutative geometry and physics.*
- 04/2003 BIRS Workshop on Noncommutative Geometry, Banff Centre, Banff, Canada.
- 05/2003 Clay Mathematics Institute Spring School on Noncommutative Geometry and its Applications, Vanderbilt University, Nashville, TN, USA.
Lecture series: *Some applications of noncommutative geometry to physics.*
- 09/2003 Workshop on Mathematical and Physical Aspects of Quantum Field Theory, Heinrich-Fabri-Institut, Blaubeuren, Germany.
Talk: *Moyal quantization and noncommutative geometry.*
- 09/2003 Noncommutative Geometry Workshop, Institut Mittag-Leffler, Djursholm, Sweden.
Talk: *Moyal algebras yield a noncompact spectral triple.*
- 10/2003 Workshop on Geometric Integral Transforms, SISSA, Trieste, Italy.
Talk: *The noncommutative local index formula for the Podleś q -sphere.*
- 01–02/2004 Winter School on Noncommutative Geometry and Mathematical Physics, CIRM, Marseille, France.
Lecture series: *Examples of noncommutative geometrical spaces.*
- 03/2004 5ème Rencontre Mathématique et Physique Lyon–Clermont, Clermont-Ferrand, France.
Talk: *Moyal-type spectral triples.*
- 04/2005 Noncommutative Geometry Spring Marathon, Uniwersytet Jagielloński, Kraków, Poland.
Talk: *Spectral manifolds: reconstruction from noncommutative geometry.*
- 01/2006 III Symposium on Quantum Information and Engineering, Wrocław, Poland.
- 04/2006 BIRS Workshop on Noncommutative Geometry, Banff Centre, Banff, Canada.
- 10/2007 Workshop on Noncommutative Manifolds II, ICTP, Trieste, Italy.
Talk: *From commutative spectral triples to manifolds.*
- 03/2010 Costa Rica Mathematica Conference 2010, San José, Costa Rica.
Talk: *Exploraciones con Mathematica: Perspectivas de un usuario.*
- 09/2010 Conference on Geometry and Physics in Cracow, Kraków, Poland.
Talk: *The spectral action in noncommutative geometry.*
- 10/2010 Mini-school on Noncommutative Geometry, IMPAN, Warszawa, Poland.
Lecture series: *Dirac operators in noncommutative geometry.*
- 09/2011 QFEXT11: Quantum Field Theory under the Influence of External Conditions, Benasque, Spain.
- 09/2011 Workshop on Physical Applications of Noncommutative Geometry, ICMAT, Madrid, Spain.

- Talk: *Riemannian manifolds in the noncommutative way.*
- 08/2012 CANP Conference: Construcción de Capacidades en Matemáticas y Educación Matemática, San José, Costa Rica.
Lecture series: *Tres caminos hacia la geometría elemental.*
- 08/2013 Mathematical Congress of the Americas 2013, Guanajuato, México.
Session Organizer: *SS#28: Noncommutative Geometry.*
Talk: *Noncommutative distances on the sphere.*
- 02/2014 Escuela de Verano EMALCA 2014, Turrialba, Costa Rica.
Talk: *La trigonometría en planos finitos.*
- 10/2016 Noncommutative Index Theory, IMPAN, Warszawa, Poland.
- 12/2016 Congreso Latinoamericano de Probabilidad y Estadística Matemática, San José, Costa Rica.
- 03/2017 Noncommutative Geometry and Applications, ICTP, Trieste, Italy.
- 07/2017 Mathematical Congress of the Americas 2017, Montréal, Canada.
Talk: *How does chirality of the Standard Model arise?*
- 07/2018 International Congress on Mathematical Physics 2018, Montréal, Canada.
Talk: *String-localized quantization: retrospect and prospect.*
- 07/2018 BIRS Workshop on Physics and Mathematics of QFT, Banff Centre, Canada.
Talk: *Wigner's "continuous-spin" representations reconsidered.*
- 09/2018 2018 QSPACE Training School, Benasque, España.
Talk: *String-local quantum fields: an overview.*
- 04/2019 Symposium on Noncommutative Geometry, Nijmegen, Netherlands.
Talk: *Classical elementary systems for Wigner's particles.*
- 06/2019 Invisibles'19 Workshop: Neutrinos, Dark Matter and Dark Energy, Valencia, Spain.
- 07/2019 Summer Research Institute on Geometry and Theoretical Physics, Villa de Leyva, Colombia.
Lecture series: *Wigner's elementary particles, classical and quantum aspects.*
- 09/2019 Noncommutative Manifolds and their Symmetries, Scalea, Italia.
Talk: *Canonical and covariant treatments of Wigner particles.*

THESES DIRECTED

- 1986 José M. Gracia-Bondía, M. Sc. in Mathematics.
Thesis *Mecánica cuántica en espacios de fases: una formulación autocontenida.*
- 1988 Ileana Castillo-Arias, Lic. in Mathematics.
Thesis *Productos cuánticos en espacios de funciones analíticas.*
- 1990 Mark B. Villarino, Lic. in Mathematics.
Thesis *Sobre la exactitud de unas fórmulas aproximativas de Ramanujan.*
- 1990 Juan Félix Ávila-Herrera, Lic. in Mathematics.
Thesis *Grupos de Lie y órbitas coadjuntas cuantizables.*
- 1996 William J. Ugalde-Gómez, M. Sc. in Mathematics.
Thesis *Operadores de Dirac en fibrados de base esférica.*

THESES: JURY MEMBER

- 1993 Manuel Nuñez Araya, M. Sc. in Informatics.

- Institute Instituto Tecnológico de Costa Rica
 Thesis Análisis de modelos de simulación para el problema del cálculo de la confiabilidad en redes.
 1993 Juan Félix Ávila-Herrera, M. Sc. in Informatics.
- Institute Instituto Tecnológico de Costa Rica
 Thesis Una implementación eficiente del algoritmo de Karmarkar.
 1997 José Rosales Ortega, M. Sc. in Mathematics.
- Institute Universidad de Costa Rica
 Thesis Cuantización geométrica y cuantización de Moyal para SU(2).
 2005 Victor Gayral, Ph. D. in Mathematical Physics.
- Institute Université de Provence, Aix-Marseille I, France.
 Thesis Déformations isospectrales non compactes et théorie quantique des champs.
 2005 Christoph Stephan, Ph. D. in Physics.
- Institute Christian-Albrechts-Universität zu Kiel, Germany.
 Thesis Noncommutative geometry and the Standard Model of particle physics.
 2007 Francesco D'Andrea, Ph. D. in Mathematics.
- Institute Scuola Internazionale Superiore di Studi Avanzati, Trieste, Italy.
 Thesis Noncommutative geometry and quantum group symmetries.

OTHER MATTERS

Member of the American Mathematical Society (since 1978), of the International Association of Mathematical Physics (since 1979) and of the T_EX Users Group (since 1987).

Tutor of the Costa Rican national team at the XI Iberoamerican Mathematics Olympiad, at Guácimo, Costa Rica, September 1996.

Founding member of the Centro de Investigaciones Matemáticas y Metamatemáticas (CIMM) of the University of Costa Rica, 1997.

Associate Member of the International Centre for Theoretical Physics, Trieste, Italy, 1998–2005.

Corresponding Member, Academia Nacional de Ciencias (Costa Rica), from 2002.

University Researcher prize, Universidad de Costa Rica, 2014.

- Languages – Fluent in English and Spanish.
 – Good speaking and reading ability in Gaelic and Portuguese.
 – Good reading ability in French, German and Italian.

- Seminars I have given seminar/colloquium talks in the following institutes:
- Universities of Clermont-Ferrand and Lille, and the Centre de Physique Théorique du CNRS at Marseille, in France
 - Universities of Bielefeld, Siegen, Mainz and Göttingen, in Germany
 - Matematisk Institut at Copenhagen and Danmarks Tekniske Højskole at Lyngby, in Denmark
 - Dublin Institute of Advanced Studies and the National University of Ireland, in Ireland
 - International Centre for Theoretical Physics (ICTP), Trieste; Scuola Internazionale Superiore di Studi Avanzati (SISSA), Trieste; and Universities of Bologna, Firenze and Napoli, in Italy
 - Universities of Zaragoza and Valladolid, Autónoma and Complutense de Madrid, and the Instituto de Ciencias Matemáticas (ICMAT) at Madrid, in Spain
 - Instytut Matematyczny Polskiej Akademii Nauk (IMPAN) in Warszawa; and Universities of Warszawa and Kraków, in Poland
 - Universidade Federal do Rio de Janeiro, in Brasil
 - University of Newcastle, NSW, in Australia
 - Universities of Rochester and Washington (Seattle) and Rice University (Houston), in USA
 - Universidad Nacional Autónoma de México, CInvEstAv of the IPN, and Mathematical Institutes in Cuernavaca and Guanajuato, in México
 - and at the Universities in Costa Rica.

LIST OF PUBLICATIONS

[·] (citations given if ≥ 50) (citations = 3612, h-index = 26)

A. Books

- [1] Joseph C. Várilly, **Elementos de geometría plana**. Editorial de la Universidad de Costa Rica, San José, Costa Rica, 1988. ISBN 9977-67-078-1.
- [2] Joseph C. Várilly, **Teoría de grupos en cuantización**. CInvEstAv del Instituto Politécnico Nacional, México, 1992. <https://hdl.handle.net/10669/11331>
- [3] J. M. Gracia-Bondía, Joseph C. Várilly and H. Figueroa, **Elements of Noncommutative Geometry**. Birkhäuser, Boston, 2001. Topcite: **855** citations. <https://link.springer.com/book/10.1007/978-1-4612-0005-5>
- [4] Joseph C. Várilly, **An Introduction to Noncommutative Geometry**. EMS Series of Lecture Notes in Mathematics **4**, European Mathematical Society Publishing House, Zürich, 2006. ISBN 3-03719-024-8. Topcite: **239** citations. https://www.ems-ph.org/books/book.php?proj_nr=41&srch=series%7Celm
- [5] Joseph C. Várilly, **Elementos de geometría plana**, Second edition. Editorial de la Universidad de Costa Rica, San José, Costa Rica, 2014. ISBN 978-9968-46-421-5. <https://www.editorial.ucr.ac.cr/ciencias-naturales-y-exactas/item/1844-elementos-de-geometria-plana.html>

B. Articles

- [6] G. G. Emch and Joseph C. Várilly, On the standard form of the Bloch equation. *Letters in Mathematical Physics* **3** (1979), 113–116. <https://dx.doi.org/10.1007/BF00400065>
- [7] G. G. Emch and Joseph C. Várilly, Some remarks on dilating semigroups of completely positive mappings. *Reports in Mathematical Physics* **18** (1980), 97–102. [https://dx.doi.org/10.1016/0034-4877\(80\)90042-7](https://dx.doi.org/10.1016/0034-4877(80)90042-7)
- [8] Joseph C. Várilly, Dilation of a non-quasifree dissipative evolution. *Letters in Mathematical Physics* **5** (1981), 113–116. <https://dx.doi.org/10.1007/BF00403239>
- [9] Joseph C. Várilly and J. M. Gracia-Bondía, The Wigner transformation is of finite order. *Journal of Mathematical Physics* **28** (1987), 2390–2392. <https://dx.doi.org/10.1063/1.527776>
- [10] J. M. Gracia-Bondía and Joseph C. Várilly, Nonnegative mixed states in Weyl–Wigner–Moyal theory. *Physics Letters A* **128** (1988), 20–24. [https://dx.doi.org/10.1016/0375-9601\(88\)91035-3](https://dx.doi.org/10.1016/0375-9601(88)91035-3)
- [11] J. M. Gracia-Bondía and Joseph C. Várilly, Algebras of distributions suitable for phase-space quantum mechanics I. *Journal of Mathematical Physics* **29** (1988), 869–879. Topcite: **221** citations. <https://dx.doi.org/10.1063/1.528200>
- [12] Joseph C. Várilly and J. M. Gracia-Bondía, Algebras of distributions suitable for phase-space quantum mechanics. II. Topologies on the Moyal algebra. *Journal of Mathematical Physics* **29** (1988), 880–887. Topcite: **111** citations. <https://dx.doi.org/10.1063/1.527984>
- [13] J. M. Gracia-Bondía and Joseph C. Várilly, Phase-space representation for Galilean quantum particles of arbitrary spin. *Journal of Physics A* **21** (1988), L879–L883. Topcite: **54** citations. <https://dx.doi.org/10.1088/0305-4470/21/18/002>
- [14] Joseph C. Várilly and J. M. Gracia-Bondía, The Moyal representation for spin. *Annals of Physics* **190** (1989), 107–148. Topcite: **256** citations. [https://dx.doi.org/10.1016/0003-4916\(89\)90262-5](https://dx.doi.org/10.1016/0003-4916(89)90262-5)
- [15] M. Gadella, J. M. Gracia-Bondía, L. M. Nieto and Joseph C. Várilly, Quadratic Hamiltonians in phase space quantum mechanics. *Journal of Physics A* **22** (1989), 2709–2738. <https://dx.doi.org/10.1088/0305-4470/22/14/021>
- [16] R. Estrada, J. M. Gracia-Bondía and Joseph C. Várilly, On asymptotic expansions of twisted products. *Journal of Mathematical Physics* **30** (1989), 2789–2796. Topcite: **89** citations. <https://dx.doi.org/10.1063/1.528514>
- [17] J. F. Cariñena, J. M. Gracia-Bondía and Joseph C. Várilly, Relativistic quantum kinematics in the Moyal representation. *Journal of Physics A* **23** (1990), 901–933. Topcite: **81** citations. <https://dx.doi.org/10.1088/0305-4470/23/6/015>
- [18] Joseph C. Várilly, J. M. Gracia-Bondía and W. Schempp, The Moyal representation of quantum mechanics and special function theory. *Acta Applicanda Mathematicae* **18** (1990), 225–250. <https://dx.doi.org/10.1007/BF00049127>

- [19] H. Figueroa, J. M. Gracia-Bondía and Joseph C. Várilly, Moyal quantization with compact symmetry groups and noncommutative harmonic analysis. *Journal of Mathematical Physics* **31** (1990), 2664–2671. Topcite: **55** citations. <https://dx.doi.org/10.1063/1.528967>
- [20] J. F. Cariñena, J. M. Gracia-Bondía, L. A. Ibort, C. López and Joseph C. Várilly, Distinguished Hamiltonian theorem for homogeneous symplectic manifolds. *Letters in Mathematical Physics* **23** (1991), 35–43. <https://dx.doi.org/10.1007/BF01811292>
- [21] Joseph C. Várilly and J. M. Gracia-Bondía, S-matrix from the metaplectic representation. *Modern Physics Letters A* **7** (1992), 659–667. <https://dx.doi.org/10.1142/S021773239200063X>
- [22] Joseph C. Várilly and J. M. Gracia-Bondía, Connes' noncommutative differential geometry and the Standard Model. *Journal of Geometry and Physics* **12** (1993), 223–301. Topcite: **191** citations. [https://dx.doi.org/10.1016/0393-0440\(93\)90038-G](https://dx.doi.org/10.1016/0393-0440(93)90038-G)
- [23] J. M. Gracia-Bondía and Joseph C. Várilly, QED in external fields from the spin representation. *Journal of Mathematical Physics* **35** (1994), 3340–3367. <https://dx.doi.org/10.1063/1.530472>
- [24] J. M. Gracia-Bondía and Joseph C. Várilly, From geometric quantization to Moyal quantization. *Journal of Mathematical Physics* **36** (1995), 2691–2701. <https://dx.doi.org/10.1063/1.531059>
- [25] C. P. Martín, J. M. Gracia-Bondía and Joseph C. Várilly, The Standard Model as a noncommutative geometry: the low energy regime. *Physics Reports* **294** (1998), 363–406. Topcite: **183** citations. [https://dx.doi.org/10.1016/S0370-1573\(97\)00053-7](https://dx.doi.org/10.1016/S0370-1573(97)00053-7)
- [26] R. Estrada, J. M. Gracia-Bondía and Joseph C. Várilly, On summability of distributions and spectral geometry. *Communications in Mathematical Physics* **191** (1998), 219–248. Topcite: **62** citations. <https://dx.doi.org/10.1007/s002200050266>
- [27] H. Figueroa, J. M. Gracia-Bondía, F. Lizzi and Joseph C. Várilly, A nonperturbative form of the spectral action principle in noncommutative geometry. *Journal of Geometry and Physics* **26** (1998), 329–339. [https://dx.doi.org/10.1016/S0393-0440\(97\)00062-4](https://dx.doi.org/10.1016/S0393-0440(97)00062-4)
- [28] Joseph C. Várilly and J. M. Gracia-Bondía, On the ultraviolet behaviour of quantum fields over noncommutative manifolds. *International Journal of Modern Physics A* **14** (1999), 1305–1323. Topcite: **113** citations. <https://dx.doi.org/10.1142/S0217751X99000671>
- [29] J. F. Cariñena, J. Clemente-Gallardo, E. Follana, J. M. Gracia-Bondía, A. Rivero and Joseph C. Várilly, Connes' tangent groupoid and strict quantization. *Journal of Geometry and Physics* **32** (1999), 79–96. [https://dx.doi.org/10.1016/S0393-0440\(98\)00028-X](https://dx.doi.org/10.1016/S0393-0440(98)00028-X)
- [30] Joseph C. Várilly, Quantum symmetry groups of noncommutative spheres. *Communications in Mathematical Physics* **221** (2001), 511–523. Topcite: **57** citations. <https://dx.doi.org/10.1007/s002200100490>
- [31] V. Gayral, J. M. Gracia-Bondía, B. Iochum, T. Schücker and Joseph C. Várilly, Moyal planes are spectral triples. *Communications in Mathematical Physics* **246** (2004), 569–623. Topcite: **192** citations. <https://dx.doi.org/10.1007/s00220-004-1057-z>
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