

Europass Curriculum Vitae

Personal information

Surname(s) / First name(s)
Address(es)
Telephone(s)
Email(s)
Nationality(-ies)
Date of birth

Joseph C. Várilly

Universidad de Costa Rica, San José 11501, Costa Rica
(+506) 2235-6578
joseph.varilly@ucr.ac.cr
Irish. Resident of Costa Rica
20 February 1952

Current position

Institute
Rank

Universidad de Costa Rica, Escuela de Matemática
Profesor Catedrático

Academic Degrees

10/1972
10/1973
05/1975
05/1980

B. Sc. in Mathematical Science, University College, Dublin
M. Sc. in Mathematics, National University of Ireland
M. A. in Mathematics, University of Rochester, NY, USA
Ph. D. in Mathematics, University of Rochester, NY, USA

Professional employment

1977–78
1979–82
1982–84
1984–87
1987–91
1991
1991–present
1998–2005
2005
2007
2011

Instructor, University of Rochester
Visiting Professor, Universidad de Costa Rica
Invited Professor, Universidad de Costa Rica
Assistant Professor, Universidad de Costa Rica
Associate Professor, Universidad de Costa Rica
Visiting Professor, CInvEstAv del Instituto Politécnico Nacional, México, DF
Full Professor (Catedrático), Universidad de Costa Rica
Associate Member, International Centre for Theoretical Physics, Trieste, Italy
Visiting Professor, Uniwersytet Warszawski, Warsaw, Poland
Visiting Professor, Universidad Complutense de Madrid
Visiting Professor, Universidad de Zaragoza

Summary of research activities

Fields
1977–82
1983–95
1992–98

(with some overlaps):
Operator algebras and applications.
Phase-space quantum mechanics and applications.
Noncommutative geometry and the Standard Model of particle physics.

1998–2013
2009–17
2010–12

Mathematical aspects of noncommutative geometry.
Quantum fields and causal renormalization.
Quantum chemistry.

Brief chronology of scientific career

- My doctoral thesis and early work centred on problems in quantum statistical mechanics. Subsequently I have worked on problems in phase-space quantization (1985–91 and 2010–2012) and on noncommutative geometry and its physical applications (since 1992).
- I have also done some collaboration in marine biology modelling (1987). On the teaching side, I have written a geometry text (1988, revised and enlarged in 2014) addressed to the needs of future mathematics teachers in Costa Rica, a monograph on Moyal quantization (1992), a graduate-level book on noncommutative geometry (2001, with two coauthors) and an introduction to noncommutative geometry (2006).
- In the area of quantization, I have studied the structure and applications of the Moyal or quantum product in phase-space. In the early nineties, I turned to noncommutative geometry (in the line of Connes), with emphasis on its applications in particle physics and its behaviour under quantum symmetries. For a time, I retook phase-space methods with a view towards quantum chemistry. Recently, I have been working on causal renormalization in quantum field theory.
- I have given courses of lectures on noncommutative geometry (NCG) at several summer schools: at Monsaraz, Portugal (the European Mathematical Society Summer School on NCG) in September 1997; at São Paulo (the X Jorge André Swieca Summer School on Particles and Fields) in February 1999; at Villa de Leyva, Colombia (the CIMPA Summer School on Geometric and Topological Methods in Quantum Field Theory) in July 2001; at Nashville (the Clay Mathematics Institute Spring School on NCG and its Applications), in May 2003; and in Warsaw (as a Marie Curie TOK lecturer) during the fall semester of 2005–06.

Research visits abroad

04/1976 – 12/1976 Subject	Instituto de Matematica, Universidade Estadual de Campinas, Brasil Infinite-dimensional holomorphy
11/1985 – 12/1985 Subject	International Centre for Theoretical Physics, Trieste, Italy Lie groups
01/1986 Subject	Departamento de Física Teórica, Universidad de Valladolid, Spain Phase-space quantum mechanics
11/1988 – 12/1988 Subject	Forschungszentrum BiBoS, Universität Bielefeld, Germany Phase-space quantum mechanics
06/1989 Subject	Departamento de Física Teórica, Universidad de Zaragoza, Spain Phase-space quantum mechanics
07/1991 – 02/1992 Subject	Centro de Investigación y Estudios Avanzados, IPN, México (on sabbatical) Phase-space quantum mechanics, noncommutative geometry
02/1993 – 03/1993 Subject	Forschungszentrum BiBoS, Universität Bielefeld, Germany Noncommutative geometry
01/1997 – 03/1997 Subject	Centre de Physique Théorique du CNRS, Marseille, France Noncommutative geometry and fundamental interactions

(of at least three weeks duration)

02/1999 Subject	Dipartimento di Matematica, Università di Bologna, Italy Noncommutative geometry
11/1999 Subject	International Centre for Theoretical Physics, Trieste, Italy Differential geometry
07/2000 – 08/2000 Subject	International Centre for Theoretical Physics, Trieste, Italy Noncommutative geometry
05/2001 Subject	Department of Mathematics, University of California, Berkeley, CA, USA Noncommutative geometry
09/2001 – 11/2001 Subject	International Centre for Theoretical Physics, Trieste, Italy Noncommutative geometry
02/2003 Subject	Dipartimento di Fisica, Università di Napoli Federico II, Napoli, Italy Noncommutative geometry and fundamental interactions
07/2003 – 10/2003 Subject	International Centre for Theoretical Physics, Trieste, Italy (on sabbatical) Noncommutative geometry and quantum symmetries
10/2003 – 12/2003 Subject	Institut des Hautes Études Scientifiques, Bures-sur-Yvette, France Noncommutative geometry
02/2004 Subject	Centre de Physique Théorique du CNRS, Marseille, France Noncommutative geometry
03/2004 Subject	Institut für Physik, Universität Mainz, Germany Noncommutative geometry and quantum symmetries
04/2004 – 05/2004 Subject	Fakultät für Physik, Universität Bielefeld, Germany Noncommutative geometry and quantum symmetries
07/2004 Subject	Department of Mathematics, University of Newcastle, NSW, Australia Noncommutative geometry
02/2005 – 04/2005 Subject	International Centre for Theoretical Physics, Trieste, Italy Noncommutative geometry and quantum symmetries
05/2005 Subject	Centre de Physique Théorique du CNRS, Marseille, France Noncommutative geometry
10/2005 – 01/2006 Subject	Katedra Metod Matematycznych Fizyki, University of Warsaw, Poland Dirac operators in noncommutative geometry
02/2007 – 04/2007 Subject	Departamento de Física Teórica, Universidad Complutense de Madrid, Spain Noncommutative geometry and quantum field theory
04/2008 Subject	Katedra Metod Matematycznych Fizyki, University of Warsaw, Poland Noncommutative geometry
10/2009 Subject	Institute of Mathematical Sciences, Australian National University, Canberra Noncommutative geometry
10/2010 Subject	Instytut Matematycznych Polskiej Akademii Nauk, Warsaw, Poland Dirac operators in noncommutative geometry
09/2011 – 12/2011 Subject	Departamento de Física Teórica, Universidad de Zaragoza, Spain (sabbatical) Quantum chemistry, quantum field theory
04/2013 – 05/2013 Subject	Fakultät für Physik, Universität Bielefeld, Germany Quantum field theory and causal renormalization

11/2015
Subject

Mathematisches Forschungszentrum Oberwolfach, Germany
Quantum field theory and causal renormalization

Scientific conference participation

- 05/1973 Conference on Infinite-Dimensional Holomorphy, Lexington, KY, USA.
05/1979 Conference on Operator Algebras, Rochester, NY, USA.
11/1980 III Costa Rican Physics Congress, San José, Costa Rica.
Talk: *Semigrupos dinámicos y ecuaciones de Bloch en sistemas abiertos finitos.*
11/1986 Workshop on Lie Groups and Representation Theory, ICTP, Trieste, Italy.
06/1989 XX International GIFT Seminar on Integrability and Quantization, Jaca, Spain.
11/1989 College on Differential Geometry, ICTP, Trieste, Italy.
Talk: *Geometric quantization on homogeneous symplectic manifolds.*
10/1990 Workshop on Differential Geometry and Mathematical Physics, CIMAT, Guanajuato, México.
Talk: *Moyal quantization on symplectic manifolds.*
03/1991 Workshop on Mathematical Physics and Geometry, ICTP, Trieste, Italy.
10/1991 Symposium on Hamiltonian Systems and Celestial Mechanics, CIMAT, Guanajuato, México.
11/1991 XXIV Congress of the Sociedad Matemática Mexicana, Oaxtepec, México.
Talk: *Representaciones proyectivas de grupos en la teoría cuántica de campos.*
06/1992 XIX International Colloquium on Group Theoretical Methods in Physics, Salamanca, Spain.
Talk: *Noncommutative geometry and the Standard Model: an overview.*
03/1993 Workshop on Mathematical Physics and Geometry, ICTP, Trieste, Italy.
Talk: *The infinite-dimensional spin representation and second quantization.*
06/1993 First Caribbean Spring School of Mathematics and Theoretical Physics, Saint-François, Guadeloupe, France.
02/1994 II Encuentro Centroamericano de Investigadores en Matemáticas, San Ramón, Costa Rica.
Talk: *La representación espín del grupo ortogonal infinitodimensional.*
03/1997 Colloque Géométrie Noncommutative et Intéractions Fondamentales, CIRM, Marseille, France.
Talk: *Spectral action from Cesàro asymptotics.*
03/1997 John Lighton Synge Centenary Conference, DIAS, Dublin, Ireland.
09/1997 EMS Summer School on Noncommutative Geometry and Applications, Monsaraz, Portugal.
Lecture series: *Introduction to noncommutative geometry.*
09/1997 Conference on Recent Results in Noncommutative Geometry, Lisboa, Portugal.
08/1998 Meeting on Noncommutative Geometry, Mathematisches Forschungszentrum Oberwolfach, Germany.
Talk: *Quantization fields over noncommutative manifolds.*
08/1998 International Congress of Mathematicians, Berlin, Germany.
Poster: *Quantum fields over noncommutative tori.*
02/1999 X Jorge André Swieca Summer School of the Sociedade Brasileira de Física, Aguas de Lindoia, SP, Brasil.

- Lecture series: *Noncommutative geometry and quantization*.
- 12/2000 XI Simposio sobre Matemáticas, Ciencia y Sociedad, San José, Costa Rica.
Talk: *La geometría en su contexto histórico*.
- 04/2001 Workshop on Quantization and Noncommutative Geometry,
Mathematical Sciences Research Institute (MSRI), Berkeley, CA, USA.
Talk: *Quantum symmetry of noncommutative spheres*.
- 05/2001 29th Canadian Annual Symposium on Operator Algebras,
Mathematical Sciences Research Institute (MSRI), Berkeley, CA, USA.
- 07/2001 CIMPA Summer School on Geometric and Topological Methods in Quantum Field
Theory, Villa de Leyva, Colombia.
Lecture series: *Cyclic cohomology, Hopf algebras and quantum theory*.
- 09/2001 Conference on Noncommutative Geometry and Quantum Groups, Banach Center,
Warszawa, Poland.
Talk: *Quantized symmetry groups of noncommutative homogeneous spaces*.
- 05/2002 VI Conference on Clifford Algebras and their Applications in Mathematical Physics,
Cookeville, TN, USA.
Invited talk: *The interface of noncommutative geometry and physics*.
- 04/2003 BIRS Workshop on Noncommutative Geometry, Banff Centre, Banff, Canada.
- 05/2003 Clay Mathematics Institute Spring School on Noncommutative Geometry and its
Applications, Vanderbilt University, Nashville, TN, USA.
Lecture series: *Some applications of noncommutative geometry to physics*.
- 09/2003 Workshop on Mathematical and Physical Aspects of Quantum Field Theory,
Heinrich-Fabri-Institut, Blaubeuren, Germany.
Talk: *Moyal quantization and noncommutative geometry*.
- 09/2003 Noncommutative Geometry Workshop, Institut Mittag-Leffler, Djursholm, Sweden.
Talk: *Moyal algebras yield a noncompact spectral triple*.
- 10/2003 Workshop on Geometric Integral Transforms, SISSA, Trieste, Italy.
Talk: *The noncommutative local index formula for the Podleś q -sphere*.
- 01–02/2004 Winter School on Noncommutative Geometry and Mathematical Physics,
CIRM, Marseille, France.
Lecture series: *Examples of noncommutative geometrical spaces*.
- 03/2004 5ème Rencontre Mathématique et Physique Lyon–Clermont,
Clermont-Ferrand, France.
Talk: *Moyal-type spectral triples*.
- 04/2005 Noncommutative Geometry Spring Marathon, Uniwersytet Jagielloński,
Kraków, Poland.
Talk: *Spectral manifolds: reconstruction from noncommutative geometry*.
- 01/2006 III Symposium on Quantum Information and Engineering, Wrocław, Poland.
- 04/2006 BIRS Workshop on Noncommutative Geometry, Banff Centre, Banff, Canada.
- 10/2007 Workshop on Noncommutative Manifolds II, ICTP, Trieste, Italy.
Talk: *From commutative spectral triples to manifolds*.
- 03/2010 Costa Rica Mathematica Conference 2010, San José, Costa Rica.
Talk: *Exploraciones con Mathematica: Perspectivas de un usuario*.
- 09/2010 Conference on Geometry and Physics in Cracow, Kraków, Poland.
Talk: *The spectral action in noncommutative geometry*.
- 10/2010 Mini-school on Noncommutative Geometry, IMPAN, Warszawa, Poland.
Lecture series: *Dirac operators in noncommutative geometry*.
- 09/2011 QFEXT11: Quantum Field Theory under the Influence of External Conditions,
Benasque, Spain.

09/2011	Workshop on Physical Applications of Noncommutative Geometry, ICMAT, Madrid, Spain. Talk: <i>Riemannian manifolds in the noncommutative way</i> .
08/2012	CANP Conference: Construcción de Capacidades en Matemáticas y Educación Matemática, San José, Costa Rica. Lecture series: <i>Tres caminos hacia la geometría elemental</i> .
08/2013	Mathematical Congress of the Americas 2013, Guanajuato, México. Session Organizer: <i>SS#28: Noncommutative Geometry</i> . Talk: <i>Noncommutative distances on the sphere</i> .
02/2014	Escuela de Verano EMALCA 2014, Turrialba, Costa Rica. Talk: <i>La trigonometría en planos finitos</i> .
10/2016	Noncommutative Index Theory, IMPAN, Warszawa, Poland.
12/2016	Congreso Latinoamericano de Probabilidad y Estadística Matemática, San José, Costa Rica.
03/2017	Noncommutative Geometry and Applications, ICTP, Trieste, Italy.

Theses directed

1986	José M. Gracia-Bondía, M. Sc. in Mathematics.
Thesis	Mecánica cuántica en espacios de fases: una formulación autocontenida.
1988	Ileana Castillo-Arias, Lic. in Mathematics.
Thesis	Productos cuánticos en espacios de funciones analíticas.
1990	Mark B. Villarino, Lic. in Mathematics.
Thesis	Sobre la exactitud de unas fórmulas aproximativas de Ramanujan.
1990	Juan Félix Ávila-Herrera, Lic. in Mathematics.
Thesis	Grupos de Lie y órbitas coadjuntas cuantizables.
1996	William J. Ugalde-Gómez, M. Sc. in Mathematics.
Thesis	Operadores de Dirac en fibrados de base esférica.

Theses: jury membership

1993	Manuel Nuñez Araya, M. Sc. en Informática.
Institute	Instituto Tecnológico de Costa Rica
Thesis	Análisis de modelos de simulación para resolver el problema del cálculo de la confiabilidad en redes.
1993	Juan Félix Ávila-Herrera, M. Sc. en Informática.
Institute	Instituto Tecnológico de Costa Rica
Thesis	Una implementación eficiente del algoritmo de Karmarkar.
1997	José Rosales Ortega, M. Sc. en Matemática.
Institute	Universidad de Costa Rica
Thesis	Cuantización geométrica y cuantización de Moyal para $SU(2)$.
2005	Victor Gayral, Ph. D. en Física Matemática.
Institute	Université de Provence, Aix-Marseille I, Francia.
Thesis	Déformations isospectrales non compactes et théorie quantique des champs.
2005	Christoph Stephan, Ph. D. en Física.
Institute	Christian-Albrechts-Universität zu Kiel, Alemania.

Thesis	Noncommutative geometry and the Standard Model of particle physics.
2007	Francesco D'Andrea, Ph. D. en Matemática.
Institute	Scuola Internazionale Superiore di Studi Avanzati, Trieste, Italia.
Thesis	Noncommutative geometry and quantum group symmetries.

Other matters

- Member of the American Mathematical Society (since 1978), of the International Association of Mathematical Physics (since 1979) and of the T_EX Users Group (since 1987).
- Tutor of the Costa Rican national team at the XI Iberoamerican Mathematics Olympiad, at Guácimo, Costa Rica, September 1996.
- Founding member of the Centro de Investigaciones Matemáticas y Metamatemáticas (CIMM) of the University of Costa Rica, 1997.
- Associate Member of the International Centre for Theoretical Physics, Trieste, Italy, 1998–2005.
- Corresponding Member, Academia Nacional de Ciencias (Costa Rica), from 2002.

Languages	Fluent in English and Spanish. Good speaking and reading ability in Gaelic and Portuguese. Good reading ability in French, German and Italian.
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Seminars	<p>I have given seminar/colloquium talks in the following institutes:</p> <p>the Universities of Clermont-Ferrand and Lille, and the Centre de Physique Théorique du CNRS at Marseille, in France;</p> <p>the Universities of Bielefeld, Siegen, Mainz and Göttingen, in Germany;</p> <p>the Matematisk Institut at Copenhagen and Danmarks Tekniske Højskole at Lyngby, in Denmark;</p> <p>the Dublin Institute of Advanced Studies and the National University of Ireland;</p> <p>the International Centre for Theoretical Physics, Trieste; the Scuola Internazionale Superiore di Studi Avanzati, Trieste; and the Universities of Bologna, Firenze and Napoli, in Italy;</p> <p>the Universities of Zaragoza and Valladolid, Autónoma and Complutense de Madrid, and the Instituto de Ciencias Matemáticas (ICMAT) at Madrid, in Spain;</p> <p>the Instytut Matematyczny Polskiej Akademiej Nauk in Warszawa; and the Universities of Warszawa and Kraków, in Poland;</p> <p>the Universidade Federal do Rio de Janeiro, in Brasil;</p> <p>the University of Newcastle, NSW, in Australia;</p> <p>the Universities of Rochester and Washington (at Seattle) and Rice University (at Houston), in the USA;</p> <p>the Universidad Nacional Autónoma de México, the CInvEstAv of the IPN, and Mathetical Institutes in Cuernavaca and Guanajuato, in México;</p> <p>and at the Universities in Costa Rica.</p>
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List of Publications

[·] (citations given if ≥ 50) (citations = 3133, h-index = 23)

A. Books

- [1] Joseph C. Várilly, **Elementos de geometría plana**. Editorial de la Universidad de Costa Rica, San José, Costa Rica, 1988. ISBN 9977-67-078-1.
- [2] Joseph C. Várilly, **Teoría de grupos en cuantización**. CInvEstAv del Instituto Politécnico Nacional, México, 1992. <http://hdl.handle.net/10669/11331>
- [3] J. M. Gracia-Bondía, Joseph C. Várilly and H. Figueroa, **Elements of Non-commutative Geometry**. Birkhäuser, Boston, 2001. Topcite: 721 citations. <http://link.springer.com/book/10.1007/978-1-4612-0005-5>
- [4] Joseph C. Várilly, **An Introduction to Noncommutative Geometry**. EMS Series of Lecture Notes in Mathematics 4, European Mathematical Society Publishing House, Zürich, 2006. ISBN 3-03719-024-8. Topcite: 214 citations. http://www.ems-ph.org/books/book.php?proj_nr=41&srch=series%7Celm
- [5] Joseph C. Várilly, **Elementos de geometría plana**, Second edition. Editorial de la Universidad de Costa Rica, San José, Costa Rica, 2014. ISBN 9968-46-421-X.

B. Articles

- [6] G. G. Emch and Joseph C. Várilly, On the standard form of the Bloch equation. *Letters in Mathematical Physics* 3 (1979), 113–116. <http://dx.doi.org/10.1007/BF00400065>
- [7] G. G. Emch and Joseph C. Várilly, Some remarks on dilating semigroups of completely positive mappings. *Reports in Mathematical Physics* 18 (1980), 97–102. [http://dx.doi.org/10.1016/0034-4877\(80\)90042-7](http://dx.doi.org/10.1016/0034-4877(80)90042-7)
- [8] Joseph C. Várilly, Dilation of a non-quasifree dissipative evolution. *Letters in Mathematical Physics* 5 (1981), 113–116. <http://dx.doi.org/10.1007/BF00403239>
- [9] Joseph C. Várilly and J. M. Gracia-Bondía, The Wigner transformation is of finite order. *Journal of Mathematical Physics* 28 (1987), 2390–2392. <http://dx.doi.org/10.1063/1.527776>
- [10] J. M. Gracia-Bondía and Joseph C. Várilly, Nonnegative mixed states in Weyl–Wigner–Moyal theory. *Physics Letters A* 128 (1988), 20–24. [http://dx.doi.org/10.1016/0375-9601\(88\)91035-3](http://dx.doi.org/10.1016/0375-9601(88)91035-3)
- [11] J. M. Gracia-Bondía and Joseph C. Várilly, Algebras of distributions suitable for phase-space quantum mechanics I. *Journal of Mathematical Physics* 29 (1988), 869–879. Topcite: 194 citations. <http://dx.doi.org/10.1063/1.528200>
- [12] Joseph C. Várilly and J. M. Gracia-Bondía, Algebras of distributions suitable for phase-space quantum mechanics. II. Topologies on the Moyal algebra. *Journal of Mathematical Physics* 29 (1988), 880–887. Topcite: 96 citations. <http://dx.doi.org/10.1063/1.527984>
- [13] J. M. Gracia-Bondía and Joseph C. Várilly, Phase-space representation for Galilean quantum particles of arbitrary spin. *Journal of Physics A* 21 (1988), L879–L883. Topcite: 52 citations. <http://dx.doi.org/10.1088/0305-4470/21/18/002>
- [14] Joseph C. Várilly and J. M. Gracia-Bondía, The Moyal representation for spin. *Annals of Physics* 190 (1989), 107–148. Topcite: 216 citations. [http://dx.doi.org/10.1016/0003-4916\(89\)90262-5](http://dx.doi.org/10.1016/0003-4916(89)90262-5)
- [15] M. Gadella, J. M. Gracia-Bondía, L. M. Nieto and Joseph C. Várilly, Quadratic Hamiltonians in phase space quantum mechanics. *Journal of Physics A* 22 (1989), 2709–2738. <http://dx.doi.org/10.1088/0305-4470/22/14/021>
- [16] R. Estrada, J. M. Gracia-Bondía and Joseph C. Várilly, On asymptotic expansions of twisted products. *Journal of Mathematical Physics* 30 (1989), 2789–2796. Topcite: 83 citations. <http://dx.doi.org/10.1063/1.528514>

- [17] J. F. Cariñena, J. M. Gracia-Bondía and Joseph C. Várilly, Relativistic quantum kinematics in the Moyal representation. *Journal of Physics A* **23** (1990), 901–933. Topcite: **73** citations. <http://dx.doi.org/10.1088/0305-4470/23/6/015>
- [18] Joseph C. Várilly, J. M. Gracia-Bondía and W. Schempp, The Moyal representation of quantum mechanics and special function theory. *Acta Applicanda Mathematicae* **18** (1990), 225–250. <http://dx.doi.org/10.1007/BF00049127>
- [19] H. Figueroa, J. M. Gracia-Bondía and Joseph C. Várilly, Moyal quantization with compact symmetry groups and noncommutative harmonic analysis. *Journal of Mathematical Physics* **31** (1990), 2664–2671. <http://dx.doi.org/10.1063/1.528967>
- [20] J. F. Cariñena, J. M. Gracia-Bondía, L. A. Ibort, C. López and Joseph C. Várilly, Distinguished Hamiltonian theorem on homogeneous complex spaces. *Letters in Mathematical Physics* **23** (1991), 35–43. <http://dx.doi.org/10.1007/BF01811292>
- [21] Joseph C. Várilly and J. M. Gracia-Bondía, S-matrix from the metaplectic representation. *Modern Physics Letters A* **7** (1992), 659–667. <http://dx.doi.org/10.1063/1.530472>
- [22] Joseph C. Várilly and J. M. Gracia-Bondía, Connes' noncommutative differential geometry and the Standard Model. *Journal of Geometry and Physics* **12** (1993), 223–301. Topcite: **186** citations. [http://dx.doi.org/10.1016/0393-0440\(93\)90038-G](http://dx.doi.org/10.1016/0393-0440(93)90038-G)
- [23] J. M. Gracia-Bondía and Joseph C. Várilly, QED in external fields from the spin representation. *Journal of Mathematical Physics* **35** (1994), 3340–3367. <http://dx.doi.org/10.1063/1.530472>
- [24] J. M. Gracia-Bondía and Joseph C. Várilly, From geometric quantization to Moyal quantization. *Journal of Mathematical Physics* **36** (1995), 2691–2701. <http://dx.doi.org/10.1063/1.531059>
- [25] C. P. Martín, J. M. Gracia-Bondía and Joseph C. Várilly, The Standard Model as a noncommutative geometry: the low energy regime. *Physics Reports* **294** (1998), 363–406. Topcite: **170** citations. [http://dx.doi.org/10.1016/S0370-1573\(97\)00053-7](http://dx.doi.org/10.1016/S0370-1573(97)00053-7)
- [26] R. Estrada, J. M. Gracia-Bondía and Joseph C. Várilly, On summability of distributions and spectral geometry. *Communications in Mathematical Physics* **191** (1998), 219–248. Topcite: **59** citations. <http://dx.doi.org/10.1007/s002200050266>
- [27] H. Figueroa, J. M. Gracia-Bondía, F. Lizzi and Joseph C. Várilly, A nonperturbative form of the spectral action principle in noncommutative geometry. *Journal of Geometry and Physics* **26** (1998), 329–339. [http://dx.doi.org/10.1016/S0393-0440\(97\)00062-4](http://dx.doi.org/10.1016/S0393-0440(97)00062-4)
- [28] Joseph C. Várilly and J. M. Gracia-Bondía, On the ultraviolet behaviour of quantum fields over noncommutative manifolds. *International Journal of Modern Physics A* **14** (1999), 1305–1323. Topcite: **107** citations. <http://dx.doi.org/10.1142/S0217751X99000671>
- [29] J. F. Cariñena, J. Clemente-Gallardo, E. Follana, J. M. Gracia-Bondía, A. Rivero and Joseph C. Várilly, Connes' tangent groupoid and strict quantization. *Journal of Geometry and Physics* **32** (1999), 79–96. [http://dx.doi.org/10.1016/S0393-0440\(98\)00028-X](http://dx.doi.org/10.1016/S0393-0440(98)00028-X)
- [30] Joseph C. Várilly, Quantum symmetry groups of noncommutative spheres. *Communications in Mathematical Physics* **221** (2001), 511–523. Topcite: **55** citations. <http://dx.doi.org/10.1007/s002200100490>
- [31] V. Gayral, J. M. Gracia-Bondía, B. Iochum, T. Schücker and Joseph C. Várilly, Moyal planes are spectral triples. *Communications in Mathematical Physics* **246** (2004), 569–623. Topcite: **156** citations. <http://dx.doi.org/10.1007/s00220-004-1057-z>

- [32] L. Dąbrowski, G. Landi, A. Sitarz, W. van Suijlekom and Joseph C. Várilly, The Dirac operator on $SU_q(2)$. *Communications in Mathematical Physics* **259** (2005), 729–759. Topcite: 89 citations.
<http://dx.doi.org/10.1007/s00220-005-1383-9>
- [33] W. van Suijlekom, L. Dąbrowski, G. Landi, A. Sitarz and Joseph C. Várilly, The local index formula for $SU_q(2)$. *K-Theory* **35** (2005), 375–394.
<http://dx.doi.org/10.1007/s10977-005-3116-4>
- [34] K. Ebrahimi-Fard, J. M. Gracia-Bondía, Li Guo and Joseph C. Várilly, Combinatorics of renormalization as matrix calculus. *Physics Letters B* **632** (2006), 552–558. <http://dx.doi.org/10.1016/j.physletb.2005.11.001>
- [35] V. Gayral, B. Iochum and Joseph C. Várilly, Dixmier traces on noncompact isospectral deformations. *Journal of Functional Analysis* **237** (2006), 507–539.
<http://dx.doi.org/10.1016/j.jfa.2006.02.010>
- [36] A. Rennie and Joseph C. Várilly, Orbifolds are not commutative geometries. *Journal of the Australian Mathematical Society* **84** (2008), 109–116.
<http://dx.doi.org/10.1017/S1446788708000293>
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